b Montrer que (I_n) est convergente

Or $0 \leqslant I_n \leqslant \frac{1}{2n+1}$ (0 ; $\frac{1}{2n+1} \to 0$)

$\lim_{n \to +\infty} \frac{1}{2n+1} = 0$

Donc $\lim_{n \to +\infty} I_n = 0$

Donc (I_n) est convergente

3 **a** Montrer que: $\forall n \in \mathbb{N}^*$, on a: $I_{n+1} + I_n = \frac{1}{2n+1}$

$$I_{n+1} + I_n = \int_0^1 \frac{u^{2n+2}}{1+u^2}\,dx + \int_0^1 \frac{u^{2n}}{1+u^2}\,dx$$

$$= \int_0^1 \frac{u^{2n+2} + u^{2n}}{1+u^2}\,du$$

$$= \int_0^1 \frac{u^{2n}(u^2+1)}{1+u^2}\,dx$$

$$= \int_0^1 u^{2n}\,dx = \left[\frac{u^{2n+1}}{2n+1}\right]_0^1$$